Portfolio Construction Theory Series - Part 3

Constrained Optimization and the Case for (or Against) Leverage

In Parts 1 and 2, we built a framework for thinking about how assets combine in a portfolio. We established that diversification benefit is maximized at the "Diversification Sweet Spot," where assets contribute equal risk, and that Sharpe ratio differences and correlation levels determine how far optimal allocations deviate from that point.

Now we apply this framework to the way investors actually make decisions. Most investors can't simply maximize Sharpe ratio unconstrained. They face a volatility ceiling that can't be exceeded, or a return floor that must be met. The key insight is to apply these concepts somewhat in reverse. Instead of asking "Given Asset #2, what is the optimal allocation between Asset #1 and Asset #2?," we want to ask, "Given the characteristics of Asset #1 and our portfolio goals, what allocation am I likely to give a second asset, and what type of asset is going to work most efficiently at that allocation?"

Maximize Portfolio Return Subject to a Maximum Volatility Constraint

If your first asset has volatility that's just a bit above your maximum allowable volatility, and if it's at least a decent Sharpe ratio standalone, you're likely to not need much of a second asset.

Start again with the S&P 500 Index ("S&P 500") (annualized total return of 14.07% (12.57% excess return) and annualized volatility of 14.04% from January 2011 through December 2025).[1] That's a good asset! You can do better on Sharpe ratio, but normally only at volatility levels that are quite a bit lower. Let's say that you feel 14.04% is just a tad too spicy volatility-wise, and you'd prefer to try and cap your portfolio volatility at 12%. It's going to be tough to find any asset that will (1) keep your volatility high (you likely want to be right at that 12% mark to maximize return) and (2) improve your Sharpe ratio because it has a higher standalone Sharpe. That means your best avenue is to focus on diversification benefit and try to reduce portfolio volatility to 12% with as little capital as possible. How do you do that? Find an asset with as low correlation as you can at as high a volatility as you can.

[1] S&P 500 excess return and volatility statistics are for the 15-year period from January 2011 through December 2025. 1.50% is the average risk-free rate over the past 15 years (January 2011 through December 2025) and is used to calculate excess return throughout this piece.

The example below shows how effective this can be:

	S&P 500	Low Correlation, High Volatility Asset #2
Expected Excess Return	12.57%	2.50%
Expected Volatility	14.04%	10.00%
Weight	84.78%	15.22%
Correlation	0.0	
Combined Excess Return	11.04%	
Combined Total Return	12.54%	
Combined Volatility	12.00%	
Sharpe Ratio	0.920	

	S&P 500	High Sharpe, but Lower Volatility and Higher Correlation Asset #2
Expected Excess Return	12.57%	4.90%
Expected Volatility	14.04%	7.00%
Weight	80.07%	19.93%
Correlation	0.5	
Combined Return	11.04%	
Combined Total Return	12.54%	
Combined Volatility	12.00%	
Sharpe Ratio	0.920	

A poor Sharpe (2.5% excess return, 10% volatility = 0.25 Sharpe) Asset #2 that hits the low correlation/high volatility mark pretty well is equally as good as a much better Sharpe (4.9% excess return, 7% volatility = 0.70 Sharpe), but lower vol and medium-correlation Asset #2.

What if we instead start with a much poorer first asset that only has excess returns of 7.50% annualized (so, 9% total return) with 14.04% volatility, which we'll call "S&P 500 with Lower Return?" We will also set a lower portfolio volatility target of 8%. Now you're probably going to have to allocate quite a bit away from the first asset, and there's meaningful room for improvement on the Sharpe ratio.

Before an example, consider what we're looking for. We need enough allocation to the second asset to try and bring portfolio volatility down to 8%, but we want that allocation to

land near the Diversification Sweet Spot. A very low volatility asset (say, 2%) has a sweet spot around 88% allocation,[2] but we'd need 50% or less to hit our 8% portfolio volatility target, leaving us far below the sweet spot and not fully capturing diversification benefits. An asset just below the target portfolio volatility (say, a 7-8% volatility asset) has a sweet spot around 64-67% allocation, but we'd likely need 80-90% to hit our 8% portfolio volatility target, pushing us past the sweet spot. A middle-ground asset (5-6% volatility) has a sweet spot around 70-75%, and the allocation needed to hit our target (60-70%) is much closer to that range. The middle ground is your most likely path to success.

The numbers bear this out. A jack-of-all-trades asset (reasonably good Sharpe, medium correlation, middle-ground volatility) produces a nice Sharpe improvement over the 0.53 Sharpe ratio of the original S&P 500 with Lower Return:

	S&P 500 Lower Return	Jack of All Trades Asset #2
Expected Excess Return	7.50%	3.05%
Expected Volatility	14.04%	6.00%
Weight	42.5%	57.5%
Correlation	0.4	
Combined Excess Return	4.94%	
Combined Total Return	6.44%	
Combined Volatility	8.00%	
Sharpe Ratio	0.618	

That asset outperforms more extreme options, including (1) a very low volatility asset with the same correlation and a far better Sharpe ratio and (2) a higher volatility asset with a better Sharpe ratio, despite the higher volatility, but higher correlation.

	S&P 500 Lower Return	Very Low Volatility Asset #2
Expected Excess Return	7.50%	1.55%
Expected Volatility	14.04%	2.00%
Weight	54.0%	46.0%
Correlation	0.4	
Combined Excess Return	4.77%	
Combined Total Return	6.27%	
Combined Volatility	8.00%	
Sharpe Ratio	0.596	

[2] Recall that the Diversification Sweet Spot is the inverse of the ratio of asset volatilities, so in this case 2%:14.04% = 88%/12% allocations at the Diversification Sweet Spot.

	S&P 500 Lower Return	Close to 8% Volatility Asset #2
Expected Excess Return	7.50%	4.30%
Expected Volatility	14.04%	7.50%
Weight	11.8%	88.2%
Correlation	0.8	
Combined Excess Return	4.68%	
Combined Total Return	6.18%	
Combined Volatility	8.00%	
Sharpe Ratio	0.585	

Minimize Volatility Subject to a Required Return

This situation is a little easier: given the expected return of Asset #1 and the expected return of potential options for Asset #2, you know exactly what allocations you'll need to hit your required return, since the portfolio return is just the weighted average of the asset returns. You then determine which of those asset allocations aligns most with the volatility-lowering principles of low correlation and equal risk sharing.

This is the exact optimization we faced with our two original hedge funds: the Low-Vol/High-Sharpe Fund #1 (2.5% excess return, 2% volatility) and the Medium-Vol/Lower-Sharpe Fund #2 (5.5% excess return, 7% volatility). To reach a portfolio that returns 10%, we know the allocations will be:

S&P 500 Allocation	Hedge Fund #1 Allocation
60%	40%

S&P 500 Allocation	Hedge Fund #2 Allocation
42%	58%

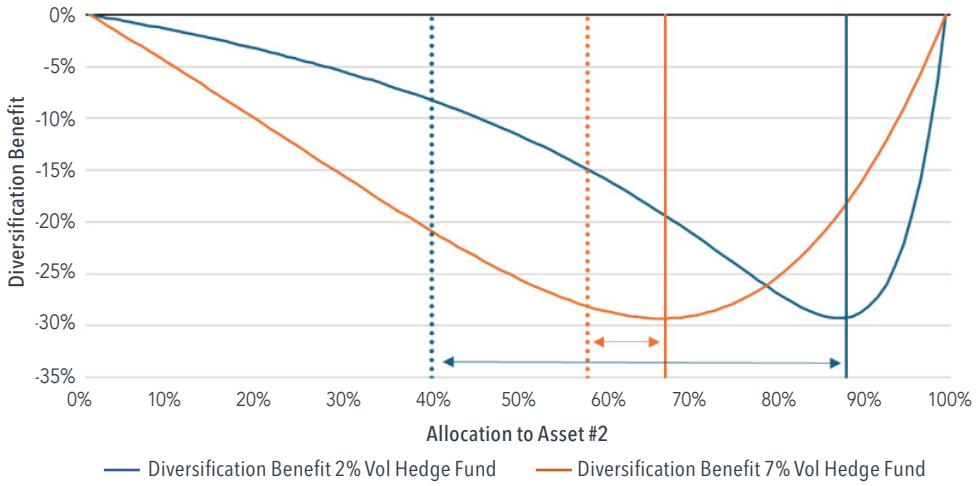
We can find the allocations that will maximize diversification benefit as well. Again, those allocations are the inverse of the ratio of volatility between the two assets. Finding that will mean each asset is contributing equal risk to the portfolio.

	S&P 500	Hedge Fund #1
Volatility	14.04%	2.00%
Inverse of Ratio of Volatilities	2.00%	14.04%
Maximum Diversification Benefit (Inverse Scaled to 100%)	12%	88%

	S&P 500	Hedge Fund #2
Volatility	14.04%	7.00%
Inverse of Ratio of Volatilities	7.00%	14.04%
Maximum Diversification Benefit (Inverse Scaled to 100%)	33%	67%

The result: when targeting a 10% total return, we end up far away from the diversification sweet spot for the Low-Vol/High-Sharpe Fund #1 (40% allocation vs 88% sweet spot) and much closer to it for the Medium-Vol/Lower-Sharpe Fund #2 (58% allocation vs 67% sweet spot).

Allocation Levels to Reach 10% Return Portfolio Have Far Different Diversification Benefits



The diversification benefit difference of almost 20% (Medium-Vol/Lower-Sharpe Fund #2 = 28.0% diversification benefit, Low-Vol/High-Sharpe Fund #1 = 8.4% diversification benefit) outweighs the fact that the Medium-Vol/Lower-Sharpe fund has a lower Sharpe ratio. The below shows the weighted average volatility of the two assets, diversification benefit, and how that results in differing portfolio volatilities:

	S&P 500	Hedge Fund #1
Expected Excess Return	12.57%	2.50%
Expected Volatility	14.04%	2.00%
Weight	60%	40%
Correlation	0.0	
Combined Excess Return	8.50%	
Combined Total Return	10.00%	
Weighted Average Volatility	9.17%	
Diversification Benefit	-8.4%	
Combined Volatility	8.40%	

	S&P 500	Hedge Fund #2
Expected Excess Return	12.57%	5.50%
Expected Volatility	14.04%	7.00%
Weight	42%	58%
Correlation	0.0	
Combined Excess Return	8.50%	
Combined Total Return	10.00%	
Weighted Average Volatility	9.99%	
Diversification Benefit	-28.0%	
Combined Volatility	7.19%	

Since the Low-Vol/High-Sharpe fund has a higher Sharpe ratio, you can make a higher Sharpe ratio portfolio with it. It just happens to be maximized at a 91% allocation to the Low-Vol/High-Sharpe fund, where you'd only have an expected total return of 4.93%. At any hedge fund allocation up to about 65%, you're better off with the Medium-Vol/Lower-Sharpe fund, because the stronger diversification benefit outweighs its lower standalone Sharpe ratio.

Sidebar: Leverage – The Full Picture

In Part 1, we noted that leverage could change the calculus. Now let's examine why that's the case, but also why the idealized case rarely holds in practice.

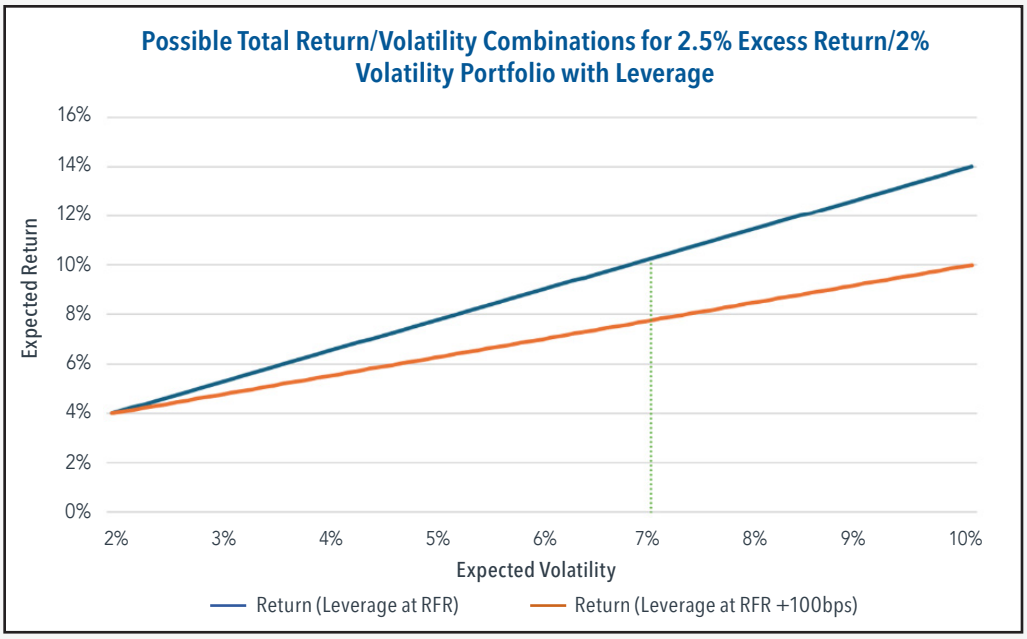
The examples above assume an unwillingness or inability to use leverage, as we've done throughout the series. But it is important to note that the math can change if an investor can obtain leverage, either directly or indirectly via a product they invest in. This is at the core of popular investment strategies and vehicles, including risk parity (apply leverage to the bond allocation to equally share risk between equities and bonds) and multi-manager platform hedge funds (find uncorrelated sources of return, even ones with low volatility, and apply leverage). This can be effective, and the potential upside is clear to see. In the most optimistic case, an investor can obtain leverage at the risk-free rate (1.50% in our case) and lever the Low-Vol/High-Sharpe fund up to 7% volatility. On the next page is the leveraged Low-Vol/High-Sharpe portfolio versus the unleveraged Medium-Vol/Lower-Sharpe portfolio, each at allocations that maximize Sharpe ratio:

	S&P 500	Hedge Fund #1 (Leverage @RFR)
Expected Excess Return	12.57%	8.75%
Expected Volatility	14.04%	7.00%
Weight	27%	73%
Correlation	0.0	
Combined Excess Return	9.77%	
Combined Total Return	11.27%	
Combined Volatility	6.35%	
Sharpe Ratio	1.54	

	S&P 500	Hedge Fund #2
Expected Excess Return	12.57%	5.50%
Expected Volatility	14.04%	7.00%
Weight	36%	64%
Correlation	0.0	
Combined Excess Return	8.06%	
Combined Total Return	9.56%	
Combined Volatility	6.77%	
Sharpe Ratio	1.19	

However, there are two issues with this idealized example. First, the leverage is obtained at a cost of the risk-free rate. In practice, borrowing costs will likely exceed the risk-free rate. Second, the linear scaling of return and volatility we have shown doesn't account for potential tail risks introduced with the use of leverage, such as margin calls.

If we instead price leverage at the risk-free rate +100bps, the Low-Vol/High-Sharpe fund is only marginally better than the Medium-Vol/Lower-Sharpe fund. Levering up the lower volatility asset to 7% volatility results in a 6.25% expected excess return, a full 2.5% worse than the 8.75% expected excess return if leverage is obtained at the risk-free rate.



Is that 75bps of extra return (6.25% for Hedge Fund #1 with leverage versus 5.5% for Hedge Fund #2) worth levering up the Low-Vol/High-Sharpe fund 3.5x? The answer requires a qualitative assessment of the risks associated with 3.5x leverage. What do the worst-case scenarios look like for the levered fund and the portfolio as a whole? What are the terms of the leverage, specifically relating to margin calls? How confident are you that the correlation between the S&P 500 and the hedge fund remains 0.0 in all market environments? These are all considerations to take into account.

There is nothing inherently bad about leverage, and its judicious use can result in increased portfolio Sharpe ratios, but one must be aware that there are diminishing returns as the cost of leverage increases and rising risks as the amount of leverage you must assume increases.

Conclusion: What We've Established

Over this three-part series, we've built a framework for thinking about how assets fit together in a portfolio:

Part 1 established that diversification benefit is maximized at the "Diversification Sweet Spot" where assets contribute equal risk to the portfolio.

Part 2 showed how Sharpe ratios and correlation interact to pull allocations away from the sweet spot.

Part 3 applied the framework to constrained optimization, showing that an asset's value depends on what allocation it will receive.

A deep understanding of the interaction between return, volatility, and correlation characteristics of candidate assets will help create better portfolios. Regardless of the structure of your individual constraints, this framework provides a clear way to think through what is most valuable in diversifying assets, and how standalone asset characteristics can be misleading in the context of building a portfolio.

We find this framework instructive, and a good check that we're directionally correct in our portfolio construction decisions, but it's nothing more than a starting point. Return, volatility, and correlation estimates are uncertain. Asset return patterns don't neatly fit a mean-variance, normal distribution-based world. Correlations shift across market regimes. These realities don't lend themselves to precise optimization. That's where judgment and experience come in. But having the right mental model for how assets interact is the foundation that judgment builds upon.

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